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General Certificate of Education

Mathematics 6360

MFP4 Further Pure 4

Mark Scheme

2006 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solutions	Marks	Total	Comments
1(a)	$\cos\theta = \dfrac{\text{scalar product}}{\text{product of moduli}}$	M1		Must be the two normals
	Scalar product in numerator = 24	B1		
	Both moduli in denominator: $\sqrt{50}$ and $\sqrt{18}$	B1		
	$\cos\theta = \dfrac{4}{5}$ or 0.8	A1	4	
(b)(i)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 5 & 3 \\ 4 & 1 & 1 \end{vmatrix} = 2\mathbf{i} + 8\mathbf{j} - 16\mathbf{k}$	M1 A1	2	Or via two scalar products, etc.
(ii)	$\mathbf{r} = \lambda\begin{bmatrix} 1 \\ 4 \\ -8 \end{bmatrix}$ or $\mathbf{r} \times \begin{bmatrix} 1 \\ 4 \\ -8 \end{bmatrix} = \mathbf{0}$	B1√ B1	2	ft d.v. (any non-zero multiple) Through *O* Must be a line equation. [Give 1/2 for correct cartesian equation.]
	Total		**8**	
2(a)	det **A** = 1	B1	1	
(b)	The z-axis $(\text{i.e } x = y = 0)$	B1	1	
(c)	Rotation about the z-axis through $\cos^{-1} 0.28$	M1 A1√ A1	3	ft axis or correct 73.7° (awrt 74°) or 1.29 rads; $\sin^{-1} 0.96$, $\tan^{-1}\dfrac{24}{7}$
	Total		**5**	
3	$\Delta = \begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2 - a^2 & c(a-b) \\ 0 & c^2 - a^2 & b(a-c) \end{vmatrix}$	M1 A1		Row/column operations (≥ 1 set) By $R_2' = R_2 - R_1$ and $R_3' = R_3 - R_1$
	$= (b-a)(c-a)\begin{vmatrix} 1 & a^2 & bc \\ 0 & a+b & -c \\ 0 & c+a & -b \end{vmatrix}$	M1 A1		Taking out factors (≥ 2 attempted)
	$= (b-a)(c-a)(c-b)\begin{vmatrix} 1 & a^2 & bc \\ 0 & a+b & -c \\ 0 & 1 & 1 \end{vmatrix}$	M1		By $R_3' = R_3 - R_2$ and then expanding fully (including attempt at all four factors)
	$= (a-b)(b-c)(c-a)(a+b+c)$ Or by use of factor theorem and cyclic symmetry	A1	6	Penalise "global" sign errors at the end only
	Total		**6**	

MFP4 (cont)

Q	Solutions	Marks	Total	Comments
4(a)(i)	The x-axis	B1	1	
(ii)	Shear (parallel to the x-axis)	M1		
	mapping e.g. $(0, 1) \to (3, 1)$ or $(1, 1) \to (4, 1)$	A1	2	Any point not on x-axis to its image
(b)	$\mathbf{A}^2 = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$	B1		
	Shear (parallel to the x-axis)	M1		
	mapping e.g. $(0, 1) \to (6, 1)$ or $(1, 1) \to (7, 1)$	A1	3	
	Total		**6**	
5(a)	$\begin{vmatrix} 1 & 3 & -1 \\ 2 & k & 1 \\ 3 & 5 & k-2 \end{vmatrix}$	M1		Attempted expansion
	$= k^2 - 2k - 10 + 9 + 3k - 5 - 6(k-2)$ $= k^2 - 5k + 6$	A1	2	AG
(b)(i)	When $k = 1$, $\Delta \neq 0 \Rightarrow \exists$ **one** soln.	B1		Give for correct single soln. $(-16, 13.5, 14.5)$ found
(ii)	When $k = 2$, $\Delta = 0$ (no unique soln.)			
	System is $x + 3y - z = 10$ $2x + 2y + z = -4$ $3x + 5y = 6$	B1		
	$R_1 + R_2 = R_3$ on *both* sides	M1		
	$\Rightarrow \exists\ \infty$-ly many soln.s	A1		
(iii)	When $k = 3$, $\Delta = 0$ (no unique soln.)			
	System is $x + 3y - z = 10$ $2x + 3y + z = -4$ $3x + 5y + z = 7$	B1		
	(1) + **(2)** $\Rightarrow 3x + 6y = 6$ **(1)** + **(3)** $\Rightarrow 4x + 8y = 17$	M1		Or $x + 2y = 2$ Or $x + 2y = 4\frac{1}{4}$ etc.
	$\Rightarrow$ System inconsistent and $\exists$ **no** soln.s	A1	7	
(c)	<u>$k = 1$</u>: the (single) point of intersection of 3 planes	B1$\checkmark$		
	<u>$k = 2$</u>: 3 planes meet in a line (or form a sheaf)	B1$\checkmark$		
	<u>$k = 3$</u>: 3 planes form a "prism" (or have three parallel lines of intersection; or have no common intersection)	B1$\checkmark$	3	ft one of each type (provided matched up correctly)
	Total		**12**	

MFP4 (cont)

Q	Solutions	Marks	Total	Comments
6(a)	Setting det **P** $(2t - 6 + 2 - 3t + 8 - 1 = 3 - t) = 0$	M1		
	$\Rightarrow t = 3$	A1	2	
(b)(i)	$\begin{bmatrix} 6 & 0 & 0 \\ -7t-21 & 3-t & 15+5t \\ 0 & 0 & 6 \end{bmatrix}$	M1		Mult^n. attempt with ≥ 3 correct elements
		A1		Any one row or column
		A1	3	All correct
(ii)	When $t = -3$, $\mathbf{PQ} = 6\mathbf{I}$	B1 B1	2	The 6 may be implicit
(iii)	$\mathbf{Q}^{-1} = \frac{1}{6}\mathbf{P} = \frac{1}{6}\begin{bmatrix} 2 & 1 & 1 \\ 1 & -3 & -2 \\ 3 & 2 & 1 \end{bmatrix}$	B1	1	Allow for $\frac{1}{6}\mathbf{P}$ or $\frac{1}{36}(6\mathbf{P})$
(c)	Enlargement (centre O) sf 6	M1 A1	2	
	Total		**10**	
7(a)	$\mathbf{a}\,.\,\mathbf{b}\times\mathbf{c} = \begin{vmatrix} 4 & -1 & 7 \\ 6 & 1 & 6 \\ 2 & 2 & -1 \end{vmatrix} = 0$	B1	1	Or noting that $\mathbf{a} + \mathbf{c} = \mathbf{b}$
(b)	$\mathbf{a}\,.\,\mathbf{c}\times\mathbf{d} = \begin{vmatrix} 4 & -1 & 7 \\ 2 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix} = 21$	M1		(Allow equivalent sets of vectors)
		m1		Expanding determinant
		A1	3	CAO (±)
(c)(i)	$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{d} - \mathbf{a})$	M1		Or equivalent attempt
	$\mathbf{r} = \begin{bmatrix} 4 \\ -1 \\ 7 \end{bmatrix} + \lambda\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} + \mu\begin{bmatrix} -3 \\ 4 \\ -9 \end{bmatrix}$	A1	2	
(ii)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & -1 \\ -3 & 4 & -9 \end{vmatrix} = -14\mathbf{i} + 21\mathbf{j} + 14\mathbf{k}$	M1		
		A1√		Any multiple ft
	$d = \begin{bmatrix} 4 \\ -1 \\ 7 \end{bmatrix}.\begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix} = 3$	M1		Must be **a**, **b**, **d** or **g** with their **n**
		A1√	4	ft for their d
(d)	$\mathbf{r} = \nu(\mathbf{a} + \mathbf{c} + \mathbf{d}) = \nu(7\mathbf{i} + 4\mathbf{j} + 4\mathbf{k})$	B1		
	Cartesian equations are $\frac{x}{7} = \frac{y}{4} = \frac{z}{4}$	B1√		ft
	$\sqrt{7^2 + 4^2 + 4^2} = 9$	M1		
	giving d.c.s $\frac{7}{9}$, $\frac{4}{9}$ and $\frac{4}{9}$	A1√	4	ft
	Total		**14**	

MFP4 (cont)

Q	Solutions	Marks	Total	Comments
8(a)(i)	Char. Equation is	M1		
	$\lambda^2 - 0\lambda + \{-a^2 - (b^2 - a^2)\} = 0$	A1		Correct to at least here
	i.e. $\lambda^2 - b^2 = 0$			
	and $\lambda = \pm b$	A1	3	
(ii)	$\lambda = b \Rightarrow (a-b)x + (a+b)y = 0$	M1		
	$\Rightarrow y = \dfrac{b-a}{b+a}x \Rightarrow$ evecs. $\alpha\begin{bmatrix} b+a \\ b-a \end{bmatrix}$	A1	2	AG
(iii)	$\lambda = -b \Rightarrow (a+b)x + (a+b)y = 0$ (etc.)	M1		
	$\Rightarrow y = -x \Rightarrow$ evecs. $\beta\begin{bmatrix} 1 \\ -1 \end{bmatrix}$	A1	2	
(b)	$\mathbf{D} = \begin{bmatrix} b & 0 \\ 0 & -b \end{bmatrix}$, $\mathbf{U} = \begin{bmatrix} b+a & 1 \\ b-a & -1 \end{bmatrix}$	B1 B1		
	and $\mathbf{U}^{-1} = \dfrac{-1}{2b}\begin{bmatrix} -1 & -1 \\ a-b & a+b \end{bmatrix}$	B1√		ft
	$\mathbf{D}^{11} = \begin{bmatrix} b^{11} & 0 \\ 0 & -b^{11} \end{bmatrix}$	B1		
	$\mathbf{M}^n = \mathbf{U}\,\mathbf{D}^n\,\mathbf{U}^{-1}$ used	M1		
	$\mathbf{M}^{11} = \dfrac{1}{2}b^{10}\begin{bmatrix} b+a & 1 \\ b-a & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ a-b & a+b \end{bmatrix}$			
	or $\dfrac{1}{2}b^{10}\begin{bmatrix} b+a & -1 \\ b-a & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ b-a & -a-b \end{bmatrix}$	A1		
	$= \dfrac{1}{2}b^{10}\begin{bmatrix} 2a & 2(a+b) \\ 2(b-a) & -2a \end{bmatrix} = b^{10}\,\mathbf{M}$	A1	7	AG
	Alternative to (a)(ii)			
	$\mathbf{M}\begin{bmatrix} b+a \\ b-a \end{bmatrix} = \begin{bmatrix} ab + a^2 + b^2 - a^2 \\ b^2 - a^2 - ab + a^2 \end{bmatrix} = b\begin{bmatrix} b+a \\ b-a \end{bmatrix}$	M1 A1		
	Alternative to (b)			
	NB $\mathbf{D}^{11} = b^{10}\,\mathbf{D}$	B1		
	Then $\mathbf{M}^{11} = \mathbf{U}\,\mathbf{D}^{11}\,\mathbf{U}^{-1}$	M2 A1		
	$= b^{10}\,\mathbf{U}\,\mathbf{D}\,\mathbf{U}^{-1} = b^{10}\,\mathbf{M}$	M2 A1		
	Total		**14**	
	TOTAL		**75**	